Written Exam Economics Summer 2018

ADVANCED MACROECONOMETRICS

June 9, 10:00 – June 11, 10:00

This exam question consists of 7 pages in total

Answers only in English. A take-home exam paper cannot exceed 20 pages plus additional 10 pages with supplementary material. One page is defined as 2400 keystrokes.

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

Be careful not to cheat at exams!
Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
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- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

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Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

1 Background Information

Regarding the data for the exam paper, please note the following:

- All assignments are based on *different* data sets. You should use the data set located in the Excel file `Data1234.xls`, where `1234` is your exam number.
- To avoid that some data sets are more difficult to handle than others, the data sets are artificial (simulated from a known data generating process), and they behave, as close as possible, like actual data.

This project examination considers empirical models for long-term growth and price formation. We have observed quite long annual time series covering a period ending in 2016 and containing the following $p = 7$ variables:

C_t	:	The log of the real private aggregate consumption in country A.
Y_t	:	The log of the real gross domestic product in country A.
π_t	:	The yearly inflation rate in country A.
C_t^*	:	The log of the real private aggregate consumption in country B.
Y_t^*	:	The log of real gross domestic product in country B.
π_t^*	:	The yearly inflation rate in country B.
OIL_t	:	The log of the index for the price of crude oil.

You are informed that the countries are located close to each other, use the same currency, and have a considerable amount of bilateral trade. The countries were to different degrees affected by the two world wars, covering the periods 1914-1917 and 1940-1945, and both countries are net oil consumers.

From an economic point of view, the candidates for equilibrium relationships include potential linkages between the real economy in the two countries, e.g.

$$Y_t = \gamma_1 + \gamma_2 Y_t^* + \gamma_3 t + u_{1t}, \tag{1.1}$$

which could reflect a common underlying development in productivity and technology. There could also be a common nominal trend driving inflation rates in the two countries,

$$\pi_t = \gamma_4 + \gamma_5 \pi_t^* + u_{2t}. \tag{1.2}$$

A different set of possible equilibrium relationships includes constant long-run propensity to consume in the two countries,

$$C_t = \gamma_6 + \gamma_7 Y_t + u_{3t} \tag{1.3}$$

$$C_t^* = \gamma_8 + \gamma_9 Y_t^* + u_{4t}, \tag{1.4}$$

where the famous great ratios would correspond to $\gamma_7 = \gamma_9 = 1$. There could also be relationships between inflation and detrended output, as in simplified Phillips curves,

$$\pi_t = \gamma_{10} + \gamma_{11}(Y_t - \gamma_{12}t) + u_{5t} \tag{1.5}$$

$$\pi_t^* = \gamma_{13} + \gamma_{14}(Y_t^* - \gamma_{15}t) + u_{6t}. \tag{1.6}$$

To be equilibrium candidates, all u_{it} variables, $i = 1, 2, ..., 6$, should be stationary.

Finally, it is unclear how the oil price variable, OIL_t, affects the two economies. Theoretically, it is likely that there could be negative supply-shock effects on real activity and consumption, and positive cost-push effects on the inflation rates.

2 The Unrestricted VAR Model

Consider a vector of variables, $x_t \in \mathbb{R}^p$, with $p \geq 2$, and the unrestricted vector autoregression (VAR):

$$x_t = \sum_{i=1}^{k} \Pi_i x_{t-i} + \phi D_t + \epsilon_t, \quad t = 1, 2, ..., T, \tag{2.1}$$

where ϵ_t is assumed to be independently Gaussian distributed, $N(0, \Omega)$, the initial values, $x_{-k+1}, ..., x_{-1}, x_0$, are considered fixed for the statistical analysis and the vector D_t contains potential deterministic variables, such as a constant, a trend, and dummy variables relevant for the empirical analysis.

[1] Assume first that the lag-length is $k = 1$. Find the moving average solution, i.e. the expression for x_t in terms of x_0, the sequence of innovations, $\{\epsilon_i\}_{i=1}^t$, and the deterministic variables, $\{D_i\}_{i=1}^t$.
Use the moving-average solution to discuss the impact of a deterministic variable, D_t, in the following three cases:

[1.1] A constant term, $D_t = 1$.

[1.2] A linear trend term, $D_t = t$.

[1.3] An impulse dummy,

$$D_t = 1(t = T_0) = \begin{cases} 1 & \text{if } t = T_0 \\ 0 & \text{otherwise.} \end{cases} \tag{2.2}$$

[2] Now assume that the lag length is $k = 2$. Show how you can use the companion form of the VAR model to find the moving average solution

$$x_t = C_0(\epsilon_t + \phi D_t) + C_1(\epsilon_{t-1} + \phi D_{t-1}) + ... + C_{t-1}(\epsilon_1 + \phi D_1) + A, \tag{2.3}$$

where A is a function of parameters and the initial values.

Calculate the first moving-average coefficients, (C_0, C_1, C_2, C_3), as functions of the coefficient matrices in (2.1).

State the conditions under which the sequence of moving-average coefficients $\{C_i\}_{i=0}^{t-1}$ will converge to zero exponentially fast.

[3] For the empirical analysis, consider the vector of variables, $x_t \in \mathbb{R}^7$, given by

$$x_t = (C_t : Y_t : \pi_t : C_t^* : Y_t^* : \pi_t^* : OIL_t)'. \tag{2.4}$$

Perform a graphical analysis of the given time series, and briefly discuss the empirical relevance of the equillibrium candidates (1.1)-(1.6) as well as the potential effects of oil prices.

[4] Set up and estimate the VAR model in (2.1) for the data in (2.4). Carefully explain the steps you take and motivate the choices you make. In particular, you should motivate your choice of deterministic variables.

Also test that the estimated model fulfills the assumptions for the unrestricted VAR model.

In practice it may not be possible to find a model that is acceptable in all directions, just do as well as you can.

3 The Cointegration Rank

[5] Now consider the error-correction form of the VAR model,

$$\Delta x_t = \Pi x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \phi D_t + \epsilon_t. \tag{3.1}$$

State $(\Pi, \Gamma_1, ..., \Gamma_{k-1})$ as functions of $(\Pi_1, ..., \Pi_k)$.

State the moving-average solution for x_t if $\text{Rank}(\Pi) = p$.

[6] Now assume that Π is restricted to have reduced rank, $\text{Rank}(\Pi) = r$, $0 < r < p$, such that

$$\Pi = \alpha\beta'. \tag{3.2}$$

State the moving average solution for this case.

Use the moving average solution to discuss the impact of a deterministic variable, D_t, in the three cases in Question 1 and compare the results.

[7] Determine the cointegration rank, r, based on your preferred statistical model using all the relevant information in the data. Be precise on the statistics you look at and their limiting distributions.

[8] Perform a recursive estimation and discuss whether the parameters of the model appear to be constant.

[9] A statistician is afraid that the long time series may contain elements of double unit roots, i.e. I(2)-ness. Explain which output from the I(1) cointegration analysis that indicate whether there may be I(2)-trends in the data you have not accounted for.
Look for sign of I(2)-ness in the empirical model.

4 Identification

[10] Impose your preferred cointegration rank and estimate the cointegrated VAR model. Test if any of the endogenous variables can be excluded from all cointegrating relationships or if they can be considered weakly exogenous.
Comment on the economic interpretation of the findings.

[11] Based on the theoretical candidates and your chosen cointegration rank, r, identify the cointegrating relationships in the empirical model. Explain the approach you use and the steps you take in the process.
Give an economic interpretation of the long-run structure.

[12] Impose any non-rejectable zero restrictions on the equilibrium adjustment parameters, α, while keeping the restrictions on β.
Interpret the equilibrium adjustment in the final model.

[13] Calculate the parameters of the Granger representation for your final model and interpret the pushing forces.
Rotate the matrices $\tilde{\beta}_\perp$ and $\alpha_\perp$ that enter the long-run impact matrix, $C = \tilde{\beta}_\perp \alpha'_\perp$, to make the interpretation as clear as possible.

5 Extension I: Time Aggregation

Consider a *quarterly* bivariate time series, $X_t \in \mathbb{R}^2$, measuring the end-of-quarter stock price index for two different markets. Assume we have observations for 25 years

$$1990:1, 1990:2, ..., 2014:4,$$

i.e. $T = 25 \cdot 4 = 100$ observations. Assume that X_t is generated by the VAR(1) model,

$$X_t = \mu + \Pi_1 X_{t-1} + \epsilon_t, \quad t = 2, ..., 100, \tag{5.1}$$

where ϵ_t is IID and Gaussian, $N(0, \Omega)$.

Consider also the time-aggregated *annual* time series, Y_τ, measuring the of end-of-year stock prices for the years

$$1990, 1991, ..., 2014,$$

such that $Y_{1990} = X_{1990:4}$, $Y_{1991} = X_{1991:4}$, ..., $Y_{2014} = X_{2014:4}$.

[14] The behavior of the annual time series, Y_τ, is also described by a VAR(1) model:

$$Y_\tau = \mu^* + \Pi_1^* Y_{\tau-1} + \epsilon_\tau^*, \quad \tau = 2, 3, ..., 25. \tag{5.2}$$

Use recursive substitution in (5.1) to find (μ^*, Π^*) as functions of (μ, Π_1). Also find the distribution of ϵ_τ^*.

[15] Now assume that the autoregressive parameter for the quarterly model is

$$\Pi_1 = \begin{pmatrix} 0.5 & 0.5 \\ 0.2 & 0.8 \end{pmatrix}, \tag{5.3}$$

such that the error-correction form of the model is

$$\Delta X_t = \mu + \Pi X_{t-1} + \epsilon_t, \tag{5.4}$$

with

$$\Pi = \Pi_1 - I_2 = \begin{pmatrix} -0.5 & 0.5 \\ 0.2 & -0.2 \end{pmatrix}. \tag{5.5}$$

Find α and β such that $\Pi = \alpha\beta'$

[16] Find the implied Π_1^* for the model for the yearly data. Also find the corresponding error correction parameter, $\Pi^* = \Pi_1^* - I_2$, and the cointegration parameters, α^* and β^*, such that $\Pi^* = \alpha^*\beta^{*\prime}$.

Comment on the results regarding the equilibrium relationships and the speed of adjustment.

6 Extension II: Common Trends

Consider two cointegrated VAR systems of each $p = 3$ variables, $X_t = (x_{1t} : x_{2t} : x_{3t})'$ and $Y_t = (y_{1t} : y_{2t} : y_{3t})'$, as given by

$$\Delta X_t = \alpha\beta' X_{t-1} + \Gamma \Delta X_{t-1} + \epsilon_t \tag{6.1}$$

$$\Delta Y_t = ab' Y_{t-1} + G \Delta Y_{t-1} + e_t. \tag{6.2}$$

You are informed that all variables are integrated of order one, $I(1)$, and the following equilibrium relationships exist:

$$x_{1t} - x_{2t} = u_{1t} \tag{6.3}$$

$$x_{2t} + \tfrac{1}{2} x_{3t} = u_{2t} \tag{6.4}$$

$$y_{1t} + y_{3t} = u_{3t}, \tag{6.5}$$

where u_{it} is a stationary process, $i = 1, 2, 3$. Each model is associated with a Granger representation. For X_t we can write

$$\begin{pmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{pmatrix} = \tilde{\beta}_\perp \sum_{i=1}^{t} \alpha_\perp' \epsilon_i + S_t + A, \tag{6.6}$$

where $\tilde{\beta}_\perp = \beta_\perp \left(\alpha'_\perp (I_p - \Gamma)\beta_\perp\right)^{-1}$ contains the loadings to the common stochastic trends, $\sum_{i=1}^{t} \alpha'_\perp \epsilon_i$, S_t is stationary process, and A is a function of initial values. Similarly for Y_t.

[17] For each model, (6.1) and (6.2), state the number of stochastic trends.
Write the loading matrices, $\tilde{\beta}_\perp$ and $\tilde{b}_\perp$, implied by the models and the information given above.

[18] Now consider a combined system of four variables, $Z_t = (x_{1t} : x_{3t} : y_{1t} : y_{2t})'$.
State the cointegrating relationships in the model for Z_t.
For the Granger representation corresponding to the model for Z_t,

$$Z_t = \beta^*_\perp \sum_{i=1}^{t} \alpha^{*\prime}_\perp \epsilon_i + S^*_t + A^*, \tag{6.7}$$

state the loading matrix to the common stochastic trends, $\beta^*_\perp$.

[19] Finally, you are informed that there exists cointegration also between the two systems in (6.1) and (6.2), such that

$$x_{3t} - y_{1t} = u_{4t}, \tag{6.8}$$

where u_{4t} is a stationary process.
State the cointegration relationships in the model for Z_t and the loading matrix for the common stochastic trends, $\beta^*_\perp$, for this case.